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AVERAGED EXPRESSION FOR THE BULK-DENSITY VECTOR OF THE CAPILLARY FORCE IN A SINTERED POWDER MIXTURE

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An expression for the surface force in a medium with developed boundary surface that is convenient for practice is obtained by an additional space averaging of the known expression for the bulk density of this force in the form of surface-energy doubled density tensor divergence.

Upon hot pressing of powders and their free sintering the powder particles are melted; this leads to the formation of a developed interface surface subjected to the action of surface-tension forces in such media [1]. In this case, a significant capillary pressure proportional to $2\Sigma/r$, where Σ is the surface-tension coefficient and r is the radius of curvature, acts on the particle surfaces. We consider the combined action of capillary forces in the processes of hot pressing and sintering within the framework of a physical model and the complete system of averaged equations of the mechanics of heterogeneous media [2]. The averaged surface-force bulk density $\boldsymbol{P}_\Sigma$ in the averaged momentum equation for the interphase boundary has the form [2, formulas (2.2.33) and (2.3.2)]

$$\boldsymbol{P}_\Sigma = s_{12}\left\langle \frac{1}{\delta' S_{12}} \int\limits_{\delta' L} \boldsymbol{\Sigma}' d'l \right\rangle_{12}, \tag{1}$$

where

$$\boldsymbol{\Sigma}' = \Sigma \boldsymbol{\tau}_{12}, \tag{2}$$

s_{12} is the interphase-surface area per unit volume, $\delta' L$ is the interphase boundary in the averaging volume dV which bounds the areas of the elements forming the interphase surface $\delta' S_{12}$ contained in the volume dV, $d'l$ is the element of length of the boundary $\delta' L$, $\langle \cdot \rangle_{12}$ is the averaging over the entire interphase surface contained in the volume dV, and $\boldsymbol{\tau}_{12}$ is the unit vector tangent to the interphase surface:

$$\boldsymbol{\tau}_{12} = \boldsymbol{l} \times \boldsymbol{n}. \tag{3}$$

Here $\boldsymbol{n}$ is the unit normal vector of the interphase surface and $\boldsymbol{l}$ is the unit vector tangent to the interphase boundary $\delta' L$. Below, we derive an expression $\langle \boldsymbol{P}_\Sigma \rangle$ that is convenient for applications by means of the additional spatial averaging (1) within the framework of the physical model from [2].

We average (1) as follows. Let dV_1 be the cube with face Δ. We choose the following $N-1$ cubes dV_m $(m = 2, 3, \ldots, N)$ in such a way that each $(m+1)$th cube contains the mth cube and the distance between the surfaces of the adjacent cubes is equal to $\delta \ll \Delta$. Figure 1 shows the boundaries of the cube faces dV_m (the solid curve) and dV_{m+1} (the dashed curve) which are perpendicular to the unit vector of the coordinate system $\boldsymbol{e}_3$, and the shaded regions are the intersection of the cube face dV_m with the melted particles contained in the powder volume. We write the averaging (1) in the form

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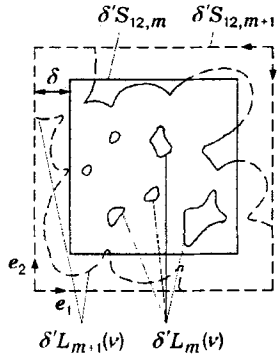

Fig. 1

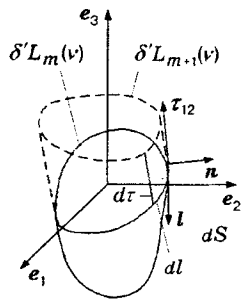

Fig. 2

$$\langle \boldsymbol{P}_\Sigma \rangle = \frac{1}{N} \sum_{m=1}^{N} \frac{1}{dV_m} \oint_{dV_m} \boldsymbol{P}_\Sigma \, dV = \frac{1}{N} \sum_{m=1}^{N} s_{12} \left\langle \frac{1}{\delta' S_{12,m}} \oint_{\delta' L_m} \boldsymbol{\Sigma}' d'l \right\rangle_{12}$$

$$= \frac{1}{N} \sum_{m=1}^{N} \frac{1}{dV_m} \sum_{\nu} \oint_{\delta' L_m(\nu)} \boldsymbol{\Sigma}' \, d'l. \tag{4}$$

Here $\delta' S_{12,m}$ is the magnitude of the interphase surface in the volume dV_m. As in [2], summation is performed over the νth boundaries $\delta' L_m(\nu)$ lying only on the cube faces dV_m and being the lines of intersection of these faces with the interphase surface. The integrals over the segments of the contours $\delta' L_m(\nu)$, which are the common boundaries of the singly connected surfaces of the multiply connected interphase boundary inside the volumes dV_m, vanish upon summation, because the unit vectors $\boldsymbol{\tau}_{12}$ of the adjacent surfaces have the opposite directions. Let

$$a = \delta N, \tag{5}$$

where $\delta \ll a \ll \Delta$. From (2)–(5), we have

$$\langle \boldsymbol{P}_\Sigma \rangle_i = \frac{1}{a} \sum_{m=1}^{N} \frac{1}{dV_m} \sum_{\nu} \delta \oint_{\delta' L_m(\nu)} \boldsymbol{\Sigma} (\boldsymbol{\tau}_{12} \cdot \boldsymbol{e}_i) \, d'l, \tag{6}$$

Here $\boldsymbol{e}_i$ is the unit vector of the orthogonal coordinate system and $\langle \boldsymbol{P}_\Sigma \rangle_i$ is the ith component of the capillary-force averaged density (4). Let the subscript g mean the cube side the normal to which is $\boldsymbol{e}_g$. We have

$$\delta = |\tau_g| \, d\tau, \tag{7}$$

where $\tau_g = \boldsymbol{\tau}_{12} \cdot \boldsymbol{e}_g$ and $d\tau$ is the distance reckoned along the interphase surface toward $\boldsymbol{\tau}_{12}$. Figure 2 shows the fragment of a powder particle contained in the cube dV_m (the solid line) and the fragment of this particle contained between the cube faces dV_m and dV_{m+1} perpendicular to $\boldsymbol{e}_3$ (the dashed line). Because $\boldsymbol{l} \cdot \boldsymbol{e}_g = 0$ on the cube face, one can decompose the vector $\boldsymbol{e}_g$ only in terms of the vectors $\boldsymbol{n}$ and $\boldsymbol{\tau}_{12}$:

$$\boldsymbol{e}_g = n_g \boldsymbol{n} + \tau_g \boldsymbol{\tau}_{12}. \tag{8}$$

Here $n_g = \boldsymbol{n} \cdot \boldsymbol{e}_g$. Multiplying (8) scalarly by $\boldsymbol{e}_i$, we obtain

$$\tau_i \tau_g = \delta_{ig} - n_i n_g, \tag{9}$$

where δ_{ig} is the Kronecker symbol. Replacing (6) dV_m by dV_1, with an error of about $a/\Delta \ll 1$, from (6), (7), and (9) we obtain

$$\langle \boldsymbol{P}_\Sigma \rangle_i = \frac{1}{a} \sum_{g=1}^{3} \frac{\Sigma}{dV_1} \left[\oint_{S^+_{12,g}} (\delta_{ig} - n_i n_g) \, d'S - \oint_{S^-_{12,g}} (\delta_{ig} - n_i n_g) \, d'S \right]. \tag{10}$$

Here $dS' = dl\,d\tau$ and $S^{+}_{12,g}$ and $S^{-}_{12,g}$ ($g = 1, 2, 3$) are the interphase surfaces contained in the volumes of the truncated pyramids whose bases are the parallel cube faces dV_1 and dV_N (the superscript plus corresponds to the truncated pyramids constructed on the sides of the above-indicated cubes having the positive coordinates relative to the center of the cube dV_1). In deriving (10), it was taken into account that $\delta = \tau_g\,d\tau$ for truncated pyramids with the plus superscript and $\delta = -\tau_g\,d\tau$ for truncated pyramids with the minus superscript.

Let

$$T_{ig} = \frac{\Sigma}{2dV} \oint\limits_{dS_{12}} (\delta_{ig} - n_i n_g)\,d'S, \tag{11}$$

where T_{ig} is the bulk-density tensor of the surface energy (the surface-energy tensor was introduced in [3]).

Since $T_{gg} = \Sigma s_{12}$ from (11), the trace of the bulk-density tensor of the surface energy is equal to the bulk density of the surface energy.

Using (11), we expand (10) in a power series of Δ, discarding terms of order Δ^2 and allowing for the fact that the volumes of the truncated pyramids are equal to approximately $a\Delta^2$. We obtain

$$\langle \boldsymbol{P}_\Sigma \rangle_i = 2\sum_{g=1}^{3} \frac{T_{ig}(t, \boldsymbol{x} + \boldsymbol{e}_g\Delta/2) - T_{ig}(t, \boldsymbol{x} - \boldsymbol{e}_g\Delta/2)}{\Delta} = 2\,\frac{\partial T_{ig}}{\partial x_g}. \tag{12}$$

Here t is the time, $\boldsymbol{x}$ are the coordinates of the center of the cube dV_1, and $\boldsymbol{x} \pm \boldsymbol{e}_g\Delta/2$ are the coordinates of the faces of this cube.

By analogy with the differential equations of motion in the continuum models with the use of stress-tensor divergence and according to (12), the tensor $2T_{ig}$ can be called a capillary-stress tensor.

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